Tabela Transformada de Laplace

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	$F(s)=\mathscr{L}\{f(t)\}$	$f(t), t>0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	1 ou $u(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{1}{s^n}, n=1,2,\ldots$	$\frac{t^{n-1}}{(n-1)!}$
5	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
6	$\frac{1}{s^{3/2}}$	$2\sqrt{\frac{t}{\pi}}$
7	$\frac{1}{s^k}, k>0$	$\frac{t^{k-1}}{\Gamma(k)}$
8	$\frac{1}{s-a}$	e^{at}
9	$\frac{1}{(s-a)^2}$	te^{at}
10	$\frac{1}{(s-a)^n}, n=1,2,\ldots$	$\frac{1}{(n-1)!}t^{n-1}e^{at}$
11	$\frac{1}{(s-a)^k}, k>0$	$\frac{1}{\Gamma(k)}t^{k-1}e^{at}$
12	$\frac{1}{(s-a)(s-b)}, a\neq b$	$\frac{1}{(a-b)}(e^{at}-e^{bt})$
13	$\frac{s}{(s-a)(s-b)}, a\neq b$	$\frac{1}{(a-b)}(ae^{at}-be^{bt})$
14	$\frac{1}{s^2+\omega^2}$	$\frac{1}{\omega}\sin\omega t$
15	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
16	$\frac{1}{s^2-a^2}$	$\frac{1}{a}\sinh at$
17	$\frac{s}{s^2-a^2}$	$\cosh at$
18	$\frac{1}{(s-a)^2+\omega^2}$	$\frac{1}{\omega}e^{at}\sin\omega t$
19	$\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos\omega t$

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	$F(s)=\mathscr{L}\{f(t)\}$	$f(t), t>0$
20	$\frac{1}{s(s^2+\omega^2)}$	$\frac{1}{\omega^2}(1-\cos\omega t)$
21	$\frac{1}{s^2(s^2+\omega^2)}$	$\frac{1}{\omega^3}(\omega t-\sin\omega t)$
22	$\frac{1}{(s^2+\omega^2)^2}$	$\frac{1}{2\omega^3}(\sin\omega t-\omega t\cos\omega t)$
23	$\frac{s}{(s^2+\omega^2)^2}$	$\frac{t}{2\omega}\sin\omega t$
24	$\frac{s^2}{(s^2+\omega^2)^2}$	$\frac{1}{2\omega}(\sin\omega t+\omega t\cos\omega t)$
25	$\frac{s}{(s^2+a^2)(s^2+b^2)}, a^2\neq b^2$	$\frac{1}{b^2-a^2}(\cos at-\cos bt)$
26	$\frac{1}{s^4+4a^4}$	$\frac{1}{4a^3}(\sin at\cosh at-\cos at\sinh at)$
27	$\frac{s}{s^4+4a^4}$	$\frac{1}{2a^2}(\sin at\sinh at)$
28	$\frac{1}{s^4-a^4}$	$\frac{1}{2a^3}(\sinh at-\sin at)$
29	$\frac{s}{s^4-a^4}$	$\frac{1}{2a^2}(\cosh at-\cos at)$
30	$\sqrt{s-a}-\sqrt{s-b}$	$\frac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
31	$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-\frac{(a+b)t}{2}}I_0\left(\frac{a-b}{2}t\right)$
32	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
33	$\frac{s}{(s-a)^{3/2}}$	$\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
34	$\frac{1}{(s^2-a^2)^k}, k>0$	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}I_{k-\frac{1}{2}}(at)$
35	$\frac{1}{s}e^{-\frac{k}{s}}$	$J_0(2\sqrt{kt})$
36	$\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\cos(2kt)$
37	$\frac{1}{s^{3/2}}e^{\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\sinh(2\sqrt{kt})$

	$F(s)=\mathscr{L}\{f(t)\}$	$f(t), t>0$
38	$e^{-k\sqrt{s}}, k>0$	$\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$
39	$\frac{1}{s}\ln s$	$-\ln t-\gamma,\ \gamma\approx 0{,}5772$
40	$\ln\frac{s-a}{s-b}$	$\frac{1}{t}(e^{bt}-e^{at})$
41	$\ln\frac{s^2+\omega^2}{s^2}$	$\frac{2}{t}(1-\cos\omega t)$
42	$\ln\frac{s^2-a^2}{s^2}$	$\frac{2}{t}(1-\cosh at)$
43	$\arctan\left(\frac{\omega}{s}\right)$	$\frac{1}{t}\sin\omega t$
44	$\frac{1}{s}\operatorname{arccot} s$	$Si(t)$

Funções especiais

[a] Delta de Dirac / Função impulso: $\delta(t)=\begin{cases}\infty, & t=0\\ 0, & t\neq 0\end{cases}$

$$\delta(t-a)=\begin{cases}\infty, & t=a\\ 0, & t\neq a\end{cases},\ \delta(t)=\frac{d}{dt}u(t),\ \int_{-\infty}^{\infty}\delta(t)\,dt=1,\ \int_{-\infty}^{\infty}f(t)\delta(t-a)\,dt=f(a)$$

[b] Função de Heaviside / Função degrau unitário: $u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$

$$Ku(t-a)=\begin{cases}K, & t>a\\ 0, & t>a\end{cases},\ u(t)=\int_{-\infty}^{t}\delta(t)\,dt$$

[c] Função Gama: $\Gamma(k)=\int_0^{\infty}e^{-x}x^{k-1}dx,\ k>0$

[d] Função de Bessel modificada de ordem ν: $I_\nu(x)=\sum_{m=0}^{\infty}\frac{x^{2m+\nu}}{2^{2m+\nu}m!\,\Gamma(m+\nu+1)}$

[e] Função de Bessel de ordem zero: $J_0(x)=1-\frac{x^2}{2^2(1!)^2}+\frac{x^4}{2^4(2!)^4}-\frac{x^6}{2^6(3!)^6}+\dots$

[f] Integral Seno: $Si(t)=\int_0^t\frac{\sin x}{x}dx$

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Propriedades da Transformada de Laplace

1	*Linearidade*	$\mathscr{L}\{af(t)+bf(t)\}=a\mathscr{L}\{f(t)\}+b\mathscr{L}\{f(t)\}$
2	*Transformada da derivada*	$\mathscr{L}\{f'(t)\}=s\mathscr{L}\{f(t)\}-f(0)$ $\mathscr{L}\{f''(t)\}=s^2\mathscr{L}\{f(t)\}-sf(0)-f'(0)$ $\mathscr{L}\{f^{(n)}(t)\}=s^n\mathscr{L}\{f(t)\}-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-f^{(n-1)}(0)$
3	*Transformada da Integral*	$\mathscr{L}\{\int_0^t f(\tau)d\tau\}=\frac{F(s)}{s}$
4	*Deslocamento no eixo s*	$\mathscr{L}\{e^{at}f(t)\}=F(s-a)$
5	*Deslocamento no eixo t*	$\mathscr{L}\{f(t-a)u(t-a)\}=e^{-as}F(s)$
6	*Mudança de escala*	$\mathscr{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right), a>0$
7	*Derivada da Transformada*	$\mathscr{L}\{tf(t)\}=-\frac{d}{ds}F(s)$ $\mathscr{L}\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}F(s), n=1,2,\ldots$
8	*Integral da Transformada*	$\mathscr{L}\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(s)ds$
9	*Transformada de Funções Periódicas*	$\mathscr{L}\{f(t)\}=\frac{1}{1-e^{sT}}\int_0^T e^{-s\tau}f(\tau)d\tau$
10	*Teorema da Convolução*	$\mathscr{L}\{f(t)*g(t)\}=F(s)G(s)$ onde $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau=g(t)*f(t)$

Teorema do Valor Inicial

$$\lim_{t\to 0^+} f(t)=\lim_{s\to\infty} sF(s)$$

Teorema do Valor Final

$$\lim_{t\to\infty} f(t)=\lim_{s\to 0^+} sF(s)$$

Transformadas Inversas de Funções Racionais Próprias

Seja $F(s)=\dfrac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}$, $a, b \in \mathbb{R}$, $m, n \in \mathbb{Z}$ e $n > m$

Então $F(s)$ pode ser expandida em uma soma de frações parciais, cujas transformadas inversas, segundo o tipo das raízes, é:

	Natureza das raízes	$F(s)=\mathscr{L}\{f(t)\}$	$f(t), t>0$
1	*Reais e distintas*	$\dfrac{K}{s+a}$	Ke^{-at}
2	*Reais e repetidas*	$\dfrac{K}{(s+a)^2}$	Kte^{-at}
3	*Complexas e distintas*	$\dfrac{K}{s+\alpha-j\beta}+\dfrac{K^*}{s+\alpha+j\beta}$	$2\|K\|e^{-\alpha t}\cos(\beta t+\theta)$
4	*Complexas e repetidas*	$\dfrac{K}{(s+\alpha-j\beta)^2}+\dfrac{K^*}{(s+\alpha+j\beta)^2}$	$2\|K\|te^{-\alpha t}\cos(\beta t+\theta)$

Nota: nos pares 1 e 2, K é uma quantidade real, ao passo que, nos pares 3 e 4, K é uma quantidade complexa $|K|e^j\theta$